

Superconducting transition temperature in thin films

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By considering the Ginzburg-Landau model, compactified in one of the spatial dimensions, and using a modified Matsubara formalism, we determine the dependence of the superconducting transition temperature (T_c) of a film as a function of its thickness (L). We show that T_c is a decreasing linear function of L^{-1} , as has been found experimentally. The critical thickness for the suppression of superconductivity is expressed in terms of the Ginzburg-Landau parameters.

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In last decades, a large amount of work has been done on the Ginzburg-Landau model applied to the study of the superconducting transition, both in the single component and in the N -component versions of the model, using the renormalization group approach. The state of the subject, for type-I and type-II superconductors and related topics, can be found for instance in Refs. [1–6]. In another related topic of investigation, there are systems that present domain walls as defects, created for instance in the process of crystal growth by some prepared circumstances. At the level of effective field theories, in many cases, this can be modeled by considering a Dirac fermionic field whose mass changes sign as it crosses the defect, meaning that the domain wall plays the role of a critical boundary separating two different states of the system [7,8]. Questions concerning stability and the existence of phase transitions may also be raised if one considers the behavior of field theories as function of spacial boundaries. Studies on confined field theory have been done in the literature since a long time ago. In particular, an analysis of the renormalization group in finite size geometries can be found in Refs. [9,10]. These studies have been performed to take into account boundary effects on scaling laws. The existence of phase transitions, would be in this case associated to some spatial parameters describing the breaking of translational invariance, for instance the distance L between planes confining the system. In this situation, for Euclidean field theories the Matsubara formalism applies for the breaking of invariance along any one of the spacial directions. Studies of this type have been recently performed [11,12], concerning with the spontaneous symmetry breaking in the $\lambda\phi^4$ theory. In particular, if one considers the Ginzburg-Landau model confined between two parallel planes, thus describing a superconducting film, the question of how the critical temperature depends on the thickness L of the film can be raised.

Under the assumption that information about general features of the behavior of superconductors, in absence of magnetic fields, can be obtained in the approximation which neglects gauge field contributions in the Ginzburg-

Landau model, in this letter we examine this model with an approach different from the renormalization group analysis. We consider the system confined between two parallel planes and we use the formalism developed in Refs. [11,12] to investigate how the critical temperature is affected by the presence of boundaries. From a physical point of view, we investigate how the critical temperature of a superconducting film depends on its thickness.

We start with the Ginzburg-Landau Hamiltonian density in the Euclidean D -dimensional space, in absence of magnetic fields, given by (in units with $\hbar = 1$)

$$\mathcal{H} = |\nabla\varphi|^2 + m_0^2 |\varphi|^2 + \frac{\lambda}{2} |\varphi|^4, \quad (1)$$

where λ is the (renormalized) self-coupling constant, with the “bare mass” given by $m_0^2 = \alpha(T - T_0)$, T_0 being the bulk transition temperature of the superconductor and $\alpha > 0$. We consider the system confined between two parallel planes, normal to the x -axis, a distance L apart from one another and use Cartesian coordinates $\mathbf{r} = (x, \mathbf{z})$, where \mathbf{z} is a $(D-1)$ -dimensional vector, with corresponding momenta $\mathbf{k} = (k_x, \mathbf{q})$, \mathbf{q} being a $(D-1)$ -dimensional vector in momenta space. The partition function is written as,

$$\mathcal{Z} = \int \mathcal{D}\varphi^* \mathcal{D}\varphi \exp \left(- \int_0^L dx \int d^{D-1}\mathbf{z} \mathcal{H}(|\varphi|, |\nabla\varphi|) \right), \quad (2)$$

with the field $\varphi(x, \mathbf{z})$ satisfying the condition of confinement along the x -axis, $\varphi(x \leq 0, \mathbf{z}) = \varphi(x \geq L, \mathbf{z}) = 0$. Then the field should have a mixed series-integral Fourier representation of the form,

$$\varphi(x, \mathbf{z}) = \sum_{n=-\infty}^{\infty} c_n \int d^{D-1}\mathbf{q} b(\mathbf{q}) e^{-i\omega_n x - i\mathbf{q}\cdot\mathbf{z}} \tilde{\varphi}(\omega_n, \mathbf{q}), \quad (3)$$

where $\omega_n = 2\pi n/L$ and the coefficients c_n and $b(\mathbf{q})$ correspond respectively to the Fourier series representation

over x and to the Fourier integral representation over the $(D-1)$ -dimensional \mathbf{z} -space. The above conditions of confinement of the x -dependence of the field to a segment of length L allow us to proceed, with respect to the x -coordinate, in a manner analogous as it is done in the imaginary-time Matsubara formalism in field theory and, accordingly, the Feynman rules should be modified following the prescription

$$\int \frac{dk_x}{2\pi} \rightarrow \frac{1}{L} \sum_{n=-\infty}^{+\infty}, \quad k_x \rightarrow \frac{2n\pi}{L} \equiv \omega_n. \quad (4)$$

We emphasize that we are considering an Euclidean field theory in D *purely* spatial dimensions, so we are *not* working in the framework of finite temperature field theory. Here, the temperature is introduced in the mass term of the Hamiltonian by means of the usual Ginzburg-Landau recipe.

To continue, we use some one-loop results described in [11,13], adapted to our present situation. These results have been obtained by the concurrent use of dimensional and *zeta*-function analytic regularizations, to evaluate formally the integral over the continuous momenta and the summation over the Matsubara frequencies. We get sums of polar (L -independent) terms plus L -dependent analytic corrections. Renormalized quantities are obtained by subtraction of the divergent (polar) terms appearing in the quantities obtained by application of the modified Feynman rules (Matsubara prescription) and dimensional regularization formulas. These polar terms are proportional to Γ -functions having the dimension D in the argument and correspond to the introduction of counter-terms in the original Hamiltonian density. In order to have a coherent procedure in any dimension, these subtractions should be performed even for those values of the dimension D where no poles of Γ -functions are present. In these cases a finite renormalization is performed.

In the following, to deal with dimensionless quantities in the regularization procedure, we introduce parameters $c^2 = m^2/4\pi^2\mu^2$, $a = (L\mu)^{-2}$, $g = 3\lambda/8\pi^2$ and $\phi_0 = \varphi_0/\mu$, where φ_0 is the normalized vacuum expectation value of the field (the classical field) and μ is a mass scale. In terms of these parameters, the one-loop contribution to effective potential, adapted to the situation under study, is given by the well known expression [14]

$$U_1(\phi, L = \infty) = \mu^D \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} g^s |\phi_0|^{2s} \int \frac{d^D k}{(k^2 + c^2)^s}, \quad (5)$$

where m (entering in c) is the renormalized mass for $L = \infty$. Performing the Matsubara replacement (4), the boundary-dependent (L -dependent) one-loop contribution to the effective potential can be written in the form

$$U_1(\phi, L) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} \frac{(-1)^s}{2s} g^s \phi_0^{2s} \times \sum_{n=-\infty}^{+\infty} \int \frac{d^{D-1} k}{(an^2 + c^2 + \mathbf{k}^2)^s}. \quad (6)$$

Now, using the dimensional regularization formula,

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + M)^s} = \frac{\Gamma(s - \frac{d}{2})}{(4\pi)^{\frac{d}{2}} \Gamma(s)} \frac{1}{M^{s - \frac{d}{2}}}, \quad (7)$$

Eq. (6) reduces to

$$U_1(\phi, L) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} f(D, s) g^s \phi_0^{2s} Z_1^{c^2}(s - \frac{D-1}{2}; a), \quad (8)$$

where $f(D, s)$ is a function proportional to $\Gamma(s - \frac{D-1}{2})$ and $Z_1^{c^2}(s - \frac{D-1}{2}; a)$ is one of the Epstein-Hurwitz *zeta*-functions, defined by

$$Z_K^{c^2}(u; \{a\}) = \sum_{n_1, \dots, n_K = -\infty}^{+\infty} \frac{1}{(a_1 n_1^2 + \dots + a_K n_K^2 + c^2)^u}, \quad (9)$$

valid for $Re(u) > K/2$ (in our case $Re(s) > D/2$).

The Epstein-Hurwitz *zeta*-function can be extended to the whole complex s -plane and we obtain, after some manipulations [15], the one-loop correction to the effective potential,

$$U_1(D, L) = \mu^D \sum_{s=1}^{\infty} g^s \phi_0^{2s} h(D, s) \times \left[2^{-(\frac{D}{2}-s+2)} \Gamma(s - \frac{D}{2}) (m/\mu)^{D-2s} + \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2 n L} \right)^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mnL) \right], \quad (10)$$

where

$$h(d, S) = \frac{1}{2^{D/2-s-1} \pi^{D/2-2s}} \frac{(-1)^{s+1}}{s \Gamma(s)} \quad (11)$$

and K_ν are the Bessel functions of the third kind.

Note that since we are using dimensional regularization techniques, there is implicit in the above formulas a factor μ^{4-D} in the definition of the coupling constant. In what follows we make explicit this factor, the symbol λ standing for the dimensionless coupling parameter (which coincides with the physical coupling constant in $D = 4$). We work in the approximation of neglecting the L -dependence of the coupling constant, that is we take λ as the *renormalized* coupling constant. In this case, it is enough for us to use only one renormalization condition,

$$\left. \frac{\partial^2}{\partial \phi^2} U_1(D, L) \right|_{\phi_0=0} = m^2 \mu^2. \quad (12)$$

Since we are using a modified minimal subtraction scheme, where the mass (and coupling constant, if it is the case) counter-terms are poles at the physical values of s , the L -dependent correction to the mass is proportional to the regular part of the analytical extension of the Epstein-Hurwitz *zeta*-function in the neighborhood of the pole at $s = 1$. Thus the L -dependent renormalized mass, at one-loop approximation, is given by

$$m^2(L) = m_0^2 + \frac{3\lambda\mu^{4-D}}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left[\frac{m}{nL} \right]^{(D-2)/2} K_{\frac{D-2}{2}}(nLm). \quad (13)$$

On the other hand, if we start in the ordered phase, the model exhibits spontaneous symmetry breaking, but for sufficiently small values of T^{-1} and L the symmetry is restored. We can define the critical curve $C(T_c, L_c)$ as the curve in the $T \times L$ plane for which the inverse squared correlation length, $\xi^{-2}(T, L, \varphi_0)$, vanishes in the L -dependent gap equation [9],

$$\xi^{-2} = m_0^2 + 6\lambda\mu^{4-D}\varphi_0^2 + \frac{6\lambda\mu^{4-D}}{L} \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{1}{\mathbf{k}^2 + \omega_n^2 + \xi^{-2}}, \quad (14)$$

where φ_0 is the normalized vacuum expectation value of the field (different from zero in the ordered phase). In the disordered phase, in particular in the neighborhood of the critical curve, φ_0 vanishes and the gap equation reduces to a L -dependent Dyson-Schwinger equation,

$$m^2(T, L) = m_0^2(T) + \frac{6\lambda\mu^{4-D}}{L} \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{1}{\mathbf{k}^2 + \omega_n^2 + m^2(T, L)}. \quad (15)$$

After steps analogous to those leading from Eq.(6) to Eq.(13), Eq.(15) can be written in the form

$$m^2(T, L) = m_0^2(T) + \frac{3\lambda\mu^{4-D}}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left[\frac{m(T, L)}{nL} \right]^{(D-2)/2} K_{\frac{D-2}{2}}(nLm(T, L)). \quad (16)$$

If we limit ourselves to the neighborhood of criticality, $m^2(T, L) \approx 0$, we may investigate the behavior of the system by using in Eq.(16) an asymptotic formula for small values of the argument of Bessel functions,

$$K_\nu(z) \approx \frac{1}{2} \Gamma(\nu) \left(\frac{z}{2} \right)^{-\nu} \quad (z \sim 0), \quad (17)$$

which allows after some straightforward manipulations, to write Eq.(16) in the form

$$m^2(T, L) \approx m_0^2(T) + \frac{3\lambda\mu^{4-D}}{(2\pi)^{D/2}} \Gamma\left(\frac{D}{2} - 1\right) L^{2-D} \zeta(D-2), \quad (18)$$

where $\zeta(D-2)$ is the Riemann *zeta*-function, $\zeta(D-2) = \sum_{n=1}^{\infty} (1/n^{D-2})$, defined for $D > 3$. Taking $m^2(T, L) = 0$ and $m_0^2 = \alpha(T - T_0)$ in Eq.(18), we obtain the critical curve in the $T \times L$ plane for Euclidean space dimension D ($D > 3$),

$$\alpha(T_c - T_0) + \frac{3\lambda\mu^{4-D}}{(2\pi)^{D/2}} \Gamma\left(\frac{D}{2} - 1\right) L_c^{2-D} \zeta(D-2) = 0. \quad (19)$$

For $D = 3$ the Riemann *zeta*-function in Eq.(19) has a pole. We can not obtain a critical curve in dimension $D \leq 3$ by a limiting procedure from Eq.(19). For $D = 3$, which corresponds to the physically interesting situation of the system confined between two parallel planes embedded in a 3-dimensional Euclidean space, Eq.(19) becomes meaningless. To obtain a critical curve in $D \leq 3$, we perform an analytic continuation of the *zeta*-function $\zeta(z)$ to values of the argument $z \leq 1$, by means of the reflection property of *zeta*-functions

$$\zeta(z) = \frac{1}{\Gamma(z/2)} \Gamma\left(\frac{1-z}{2}\right) \pi^{z-\frac{1}{2}} \zeta(1-z), \quad (20)$$

which defines a meromorphic function having only one simple pole at $z = 1$. For $D = 3$, a mass renormalization procedure can be done as follows: remembering the formula

$$\lim_{z \rightarrow 1} \left[\zeta(z) - \frac{1}{1-z} \right] = \gamma, \quad (21)$$

where $\gamma \cong 0.57216$ is the Euler constant, we define the *renormalized* mass \bar{m} as

$$\begin{aligned} \bar{m}^2(T, L) &= \lim_{D \rightarrow 3-} \left[m^2(T, L) - \frac{3\lambda\mu}{2\pi\sqrt{2}L(3-D)} \right] \\ &= \alpha(T - T_0) + \frac{3\gamma\lambda\mu}{2\sqrt{2}\pi L}. \end{aligned} \quad (22)$$

Taking this *renormalized* mass equal to zero leads to the critical curve in dimension $D = 3$, given by

$$T_c = T_0 - \frac{3\gamma\lambda\mu}{2\sqrt{2}\pi\alpha L_c}. \quad (23)$$

In Eq.(23), T_0 corresponds to the transition temperature for the material in absence of boundaries ($L_c \rightarrow \infty$), that is, to the bulk transition temperature. We see then that, in a film made of the same material, the critical temperature is diminished by a quantity proportional to the inverse of its thickness. Also, we see that there is a minimal film thickness $L_c^{(0)}$ below which superconductive is

suppressed, which is given by (identifying the Ginzburg-Landau parameter $\beta = \lambda\mu$)

$$L_c^{(0)} = \frac{3\gamma\beta}{2\sqrt{2}\pi\alpha T_0}. \quad (24)$$

Such a linear dependence of T_c with the inverse of the film thickness has been found experimentally in materials containing transition metals, for example, in Nb [17–19] and in W-Re alloys [20]; for these cases, it has been explained in terms of proximity, localization and Coulomb-interaction effects. Notice that our result does not depend on microscopic details of the material involved nor accounts for the influence of manufacturing aspects, like the kind of substrate on which the film is deposited. In other words, the linear decreasing of T_c as the film thickness is diminished emerges solely as a topological effect of the compactification of the Ginzburg-Landau model in one direction; other aspects, which may influence the transition temperature of the film, will show up experimentally as deviations of this linear behavior.

Here, in a field theoretical framework, we have shown that quantum corrections to the mass in the Ginzburg-Landau model compactified in one of the spatial dimensions, in one loop-order, leads to the linear dependence of T_c with the inverse of the thickness for a film, superconductivity being suppressed at a minimum critical thickness $L_c^{(0)}$. One expects, however, that the inclusion of the L -dependence of the coupling constant in first-order may lead to a small correction to the linear behavior obtained. Our treatment can also be extended to consider external magnetic fields, but these issues will be discussed elsewhere.

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